Dynamics of the Markov Time Scale of Seismic Activity May Provide a Short-Term Alert for Earthquakes

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We propose a novel method for analyzing precursory seismic data before an earthquake that treats them as a Markov process and distinguishes the background noise from real fluctuations due to an earthquake. A short time (on the order of several hours) before an earthquake the Markov time scale $t_M$ increases sharply, hence providing an alarm for an impending earthquake. To distinguish a false alarm from a reliable one, we compute a second quantity, $T_1$, based on the concept of extended self-similarity of the data. $T_1$ also changes strongly before an earthquake occurs. An alarm is accepted if both $t_M$ and $T_1$ indicate it simultaneously. Calibrating the method with the data for one region provides a tool for predicting an impending earthquake within that region. Our analysis of the data for a large number of earthquakes indicate an essentially zero rate of failure for the method.

Earthquakes are complex phenomena.1 Although still subject to some debate, precursory anomalies, such as changes in the seismic recordings, and anomalous variations in the chemical, hydrological, and electromagnetic properties of the area in which earthquakes occur, usually precede a large earthquake.1,2 One school of thought believes that the anomalies occur within days to weeks before the main shock, but probably not much earlier,3 and that the spatial precursory patterns develop at short distances from impending large earthquakes. A second school believes that the anomalies may occur up to decades before large earthquakes, at distances much larger than the length of the main shock rupture, a concept closely linked to the theory of critical phenomena1,2 which was advocated1,4,5 as early as 1964 with a report4 documenting the existence of long-range correlations in the precursors. Knopoff et al.6 reported recently the existence of long-range spatial correlations in the increase of medium-range magnitude seismicity prior to large earthquakes in California.

Pursuing a model of rock rupture and its relation with critical phenomena and earthquakes,7 a method of analysis was introduced8,9 that, for certain values of its parameters, led to a power law (typical of critical phenomena) for the system’s time-to-failure. Several groups10 proposed percolation11 and hierarchical models of damage/rupture prior to an earthquake. In particular, Sahimi et al.12 proposed a connection between percolation, the spatial distribution of earthquakes’ hypocenters, and rock’s fracture/fault networks. Sornette and Sammis13 developed a theory according to which the power law that describes the accelerated seismicity close to a large earthquake is accompanied by log-periodic corrections14 which were shown15 to also exist in the power law that describes the increase in the energy that rock releases as it undergoes fracturing. Such ideas were further developed by Huang et al.16 with empirical evidence provided by Bowman et al.17 and view a large earthquake as a temporal singularity in the seismic time series, resulting from the collective behavior and accumulation of many previous smaller-size events.18 In this picture, as the stress on rock increases, micro-ruptures develop that redistribute the stress and generate fluctuations in it. As damage accumulates, the fluctuations become spatially and temporally correlated, resulting in a larger number of significantly-stressed large domains. The correlations accelerate the spatial smoothing of the fluctuations, culminating in a rupture with a size on the order of the system’s size, and representing its final state in which earthquakes occur. Numerical19 and empirical20 evidence for this picture indicates that, similar to critical phenomena, the correlation length of the stress-field fluctuations increases significantly before a large earthquake. Notwithstanding the evidence, proving or refuting the notion of earthquakes as a critical phenomenon entails replacing the proxies, used for checking the build-
up of the cooperativity that leads to large earthquakes, by a direct measure of the dynamic evolution of the stress field. Unfortunately, such a procedure is far beyond the present technical abilities.

A theory of earthquakes should predict, (1) when and (2) where they occur in a wide enough region. It should also be able to (3) distinguish a false alarm from a reliable one. In this paper, we propose a method for predicting earthquakes which possesses the three features. The method estimates the Markov time scale (MTS) $t_M$ of a seismic time series - the time over which the data can be represented by a Markov process. As the seismic data evolve with the time, so also does $t_M$. We show that the time evolution of $t_M$ provides an effective alarm a short time before earthquakes. The method distinguishes abnormal variations of $t_M$ before the arrival of the P-waves, hence providing enough of a warning for triggering a damage/death-avoiding response prior to the arrival of the more damaging S-waves.

The method first checks whether the seismic data follow a Markov chain and, if so, measures the function MTS $t_M$. Characterization of the statistical properties of fluctuations of $n$ measured quantities of the stochastic process $x(t)$ requires evaluation of the joint probability distribution function (PDF) $P_n(x_1, t_1; \cdots; x_n, t_n)$. If the data are a Markov process, then $P_n = \Pi_{i=1}^{n-1} p(x_{i+1}, t_{i+1} | x_i, t_i)p(x_1, t_1)$, where $p(x_{i+1}, t_{i+1} | x_i, t_i)$ are conditional probabilities such that the Chapman-Kolmogorov (CK) equation,

$$p(x_2, t_2 | x_1, t_1) = \int dx_3 p(x_2, t_2 | x_3, t_3)p(x_3, t_3 | x_1, t_1)$$  \hspace{1cm} (1)

holds for any $t_3$ in $t_1 < t_3 < t_2$. The validity of the CK equation for describing the process is checked by comparing the directly-evaluated $p(x_2, t_2 | x_1, t_1)$ with the those calculated according to right side of Eq. (1). To determine $t_M$ for the data we compute for given $x_1$ and $x_2$ the quantity, $Q = |p(x_2, t_2 | x_1, t_1) - \int dx_3 p(x_2, t_2 | x_3, t_3)p(x_3, t_3 | x_1, t_1)|$, in terms of, for example, $t_3 - t_1$. In practice, we take $t_1 = 0$ and $t_3 = \frac{1}{2}t_2$, and vary $t_2$. Plotting $Q$ versus $t_3$ produces the position of $t_M$ in the limit $Q \rightarrow 0.25$

Our analysis of seismic data (see below) indicates that the average $t_M$ for the uncorrected background seismic time series is much smaller than that for earthquakes data (P-wave plus S-wave). Thus, at a certain time before an earthquake, $t_M$ rises significantly and provides an alarm for the earthquake. As we show below, the alert time $t_0$ is on the order of hours, and depends on the earthquake’s magnitude $M$ and the epicenter’s distance $d$ from the data-collecting station(s). The sharp rise in $t_M$ at the moment of alarm is, in some sense, similar to the increase in the correlation length $\xi$ of the stress-field fluctuations in the critical phenomena theories of earthquake, since $t_M$ is also the time over which the events leading to an earthquake are correlated.

Therefore, just as the correlation length $\xi$ increases as the catastrophic rupture develops, so also does $t_M$. However, whereas it is exceedingly difficult to directly measure $\xi$, $t_M$ is computed rather readily. Moreover, whereas $\xi$ is defined for the entire rupturing system over long times, $t_M$ is computed online (in real time), hence reflecting the correlations of the most recent events that are presumably most relevant to an impending earthquake.

To distinguish a false alarm that might be indicated by $t_M$ from a true one, we use a second time-dependent function that we compute based on the extended self-similarity (ESS) of the seismic time series. The ESS is characterized by $S_p$, a structure function of order $p$, defined by

$$S_p(\tau) = \langle |x(t + \tau) - x(t)|^p \rangle \sim \langle |x(t + \tau) - x(t)|^3 \rangle^{\frac{1}{p}} \hspace{1cm} (2)$$

where $\tau$ is the lag (in units of data points). The first nontrivial moment (beyond the average and variance) of a distribution is $S_3$, and because for a Gaussian process, $\zeta_p = \frac{1}{2}p$, the deviations from this relation represent non-Gaussian behavior. It is also well-known that the moments $S_p$ with $p < 1$ contain information on frequent events in a time series. Prior to an earthquake the number of frequent events (development of cracks that join up) suddenly rises, indicated by a sudden change in $S_p$ with $p < 1$. We observe that the starting point of $S_p(\tau)$ ($p < 1$) versus $S_3(\tau)$ is different for different type of data set. To determine the distance form the origin we define the function $T_1 = T(\tau = 1) = [S_3^2(\tau = 1) + S_3^3(\tau = 1)]^{1/2}$. Close to an earthquake the function $T_1(t)$, also estimated online, suddenly changes and provides a second alert. Its utility is due to the fact that it is estimated very accurately even with very few data points, say 50, hence enabling online analysis of the data collected over intervals of about 1 second. Thus, even with few data points, the method can detect the change of correlations in the incoming data. For example, for correlated synthetic data with a spectral density $1/f^{2\alpha-1}$, one obtains

$$T_1 = -7\alpha + 7.$$  

We have analyzed the data for vertical ground velocity $V_z(t)$ for 173 earthquakes with magnitudes $3.2 \leq M \leq 6.3$ that occurred in Iran between $28^\circ$N and $40^\circ$N latitude, and $47^\circ$E and $62.5^\circ$E longitude, between January 3 and July 26, 2004. Recorded by 14 stations, the data can be accessed at http://www.iiees.ac.ir/bank/bank2004.html. The frequency was 40 Hz for 2 of the stations and 50 Hz for the rest. The vertical ground velocity data were analyzed because with our method they provide relatively long (on the order of several hours), and hence useful, alarms for the impending earthquakes. Forty (discrete) data points/second are recorded in the broad-band seismogram for the vertical ground velocity $x(t) \equiv V_z$. To analyze such data and provide alarms for the area for which the data are analyzed, we proceed as follows.
The MTS sensors were (broad-band) Guralp CMG-3T that collect data for long-enough data series (10^3 data points or more) the function \( t_M \) are estimated where \( Q \) provides estimates of \( t_M \). (3) \( T_1(t) \) is computed for the same data. To compute \( S_p(\tau) \) (we used \( p = 1/10 \)) the data \( x(t) \) are normalized by their standard deviation, hence making \( T_1 \) dimensionless. (4) Steps (1)-(3) are repeated for a large number of previously-occurred earthquakes of size \( M \) at a distance \( d \) from the station, referred to as \((M, d)\) earthquakes. Earthquakes with \( M < M_c \) and \( d > d_c \) are of no practical importance and are ignored (we used \( M_c = 4.5 \) and \( d_c = 150 \) km). (5) Define the thresholds \( t_{Mc} \) and \( T_{1c} \) such that for \( t_M > t_{Mc} \) and \( T_1 > T_{1c} \) one has an alert for an earthquake \((M > M_c, d < d_c)\). If \( t_{Mc} \) and \( T_{1c} \) are too large no alert is obtained, whereas one may receive useless alerts if they are too small. By comparing the data for all the earthquakes with \( M > M_c \) registered in a given station, \( t_{Mc} \) and \( T_{1c} \) for the earthquakes are estimated. (6) Real-time data analysis is performed to compute the function \( t_M(t) \) and \( T_1(t) \). An alarm is turned on if \( t_M > t_{Mc} \) and \( T_1 > T_{1c} \) simultaneously. When the alarm is turned on, it indicates that an earthquake of magnitude \( M \geq M_c \) at a distance \( d \leq d_c \) is going to occur. The procedure can be carried out for any station. The larger the amount of data, the more precise the alarm will be.

Figure 1 presents \( T_1(t) \) and \( t_M(t) \) for an \( M = 6.3 \) earthquake, occurred on May 28, 2004 at 12:36 am in Baladeh at (36.37N, 51.64E, depth 28) in northern Iran. The data were collected at Karaj station (near Tehran, Iran) at a distance of 74 km from the epicenter, and a depth of 70 m. The earthquake catalogue in the internet address given above indicates that, for several days before the main event, there was no foreshock in that region. Thus, \( T_1 \) and \( t_M \) provided a seven hour alarm for the Baladeh earthquake. Since the data used for computing \( t_M \) and \( T_1 \) were, respectively, in strings of 200 and 50 points, there is no effect of the events before they were collected and, hence, the patterns in Fig. 1 reflect the events taking place in the time period in which the data were collected.

To estimate the alert times \( t_a \), which are on the order of hours, we carried out an analysis of online data for 14 stations in Iran’s broad-band network (the sensors are Guralp CMG-3T broad-band), analyzing the vertical ground velocity data. Our analysis indicates that \( t_a \) depends on \( M \), being small for low \( M \), but quite large for large \( M \). Using extensive data for the Iranian earthquakes with \( M > 4.5 \) and \( d \leq 150 \) km, we have obtained an approximate relation for the broad-band stations, shown in Figure 2 and represented by

\[
\log t_a = -1.35 + 2.4 \log M ,
\]

where \( t_a \) is in hours. The numerical coefficients of Eq. (3) for each area should be estimated from the data collected for that area. The above analysis can clearly be extended to all the stations around the world. This is currently underway for Iran’s network. For an earthquake of magnitude \( M = 4.5 \), Eq. (3) predicts an alert time of about 2 hours. Thus, if, for example, three hours after the alarm is turned on, the earthquake has not still happened, we know that the magnitude of the coming earthquake is \( M \geq 5.7 \).

In summary, we have proposed a new method for analyzing seismic data and making predictions for when an earthquake may occur with a magnitude \( M \geq M_c \) at a distance \( d \leq d_c \). The method is based on computing the Markov time scale \( t_M \), and a quantity \( T_1 \) calculated based on the concept of extended self-similarity of the data, and monitoring them online as they evolve with the time. If the two quantities exceed their respective critical thresholds \( t_{cM} \) and \( T_{1c} \), estimated based on analyzing the data for the previously-occurred earthquakes, an alarm is tuned on. We are currently utilizing this method for Iran’s stations. To do so, we calibrate the method with the data for the stations in one region (i.e., estimate \( t_{cM} \) and \( T_{1c} \) for distances \( d < d_c \). If in a given region there is a single station, then once the online-computed \( t_M \)

![FIG. 1. Time-dependence of \( T_1 \) and \( t_M \) for a recent earthquake of magnitude 6.3 in northern Iran, and their comparison with the vertical ground velocity data \( V_z(t) \). \( t_M \) is in number of data points (the frequency at the station is 40 Hz), \( T_1 \) is dimensionless, while \( V_z(t) \) is in “counts” which, when multiplied by a factor \( 1.1382 \times 10^{-3} \), is converted to \( \mu m/sec \). The sensors were (broad-band) Guralp CMG-3T that collect data in the east-west, north-south and vertical directions. The thresholds are \( t_{Mc} = 5.6 \) and \( T_{1c} = 0.88 \).](image)
and $T_1$ exceed their critical values, the alarm is turned on. If there are several stations, then once they declare that their $t_M$ and $T_1$ have exceeded their thresholds, the alarm is turned on. If after about 2 hours, no earthquake has occurred yet, then we know that the magnitude of the incoming earthquake will be greater $M_c = 4.5$ at a distance $d < d_c$.

Over the past two years, the method has been utilized in the Iranian stations. Our analysis indicates that the method’s failure rate decreases to essentially zero when $t_M$ and $T_1$ provide simultaneous alarms. That is, practically every earthquake that we have considered, including those that have been occurring while we have been performing online analysis of their incoming data and providing alarms for them (with $M > M_c$), was preceded by an alarm. Of all the earthquakes that we have analyzed so far, the method has failed in only two cases. In our experience, if after 10 hours no earthquake occurs, we count that as a failed case. However, as mentioned, we have so far had only two of such cases.

Finally, it must be pointed out that the most accurate alarms are obtained from stations that receive data from depths of $> 50$ m, and are perpendicular to the active faults that cause the earthquake, since they receive much more correlated data for the development of the cracks than any other station.

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FIG. 2. The dependence of alert time $t_a$ (in hours) on the magnitude $M$ of the earthquakes, obtained based on the data from broad-band stations by analyzing 173 earthquakes with magnitudes $3.2 \leq M \leq 6.3$ that occurred in Iran between $28^\circ$N and $40^\circ$N latitude, and $47^\circ$E and $62.5^\circ$E longitude, between January 3 and July 26, 2004.


